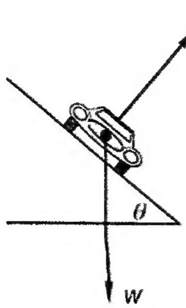
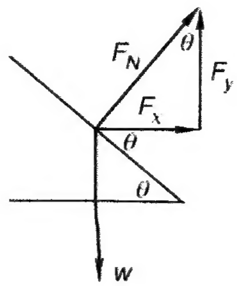
Centripetal Force in Several Settings

*6) If the roadway is banked at the proper angle for a given radius of turn and a given speed, the required centripetal force F_c can be provided by the horizontal component of the normal force and **no** frictional force will be required.*





$$F_y = mg$$

$$F_x = F_c = m\frac{v^2}{r}$$

The vertical component of F_N is F_y, whose magnitude must equal the weight, mg, so that the sum of the vertical forces is equal to zero. The horizontal component of the normal force must be the required centripetal force F_c. Thus θ is the angle whose tangent is F_x over F_y.

A car whose mass is 1400 kg is driven at a constant speed of 30.00 meters per second around a banked track whose radius is 100.0 meters. (a) What is the centripetal force exerted on the car? (b) What is F_y, the vertical component of F_N? (c) At what angle should the track be banked so that the vector sum of these forces is perpendicular to the track?

(a) $F_c = m\dfrac{v^2}{r}$ centripetal force

$= (1400\text{ kg})\dfrac{\left(30.00\ \dfrac{\text{m}}{\text{s}}\right)^2}{100.0\text{ m}}$ substituted

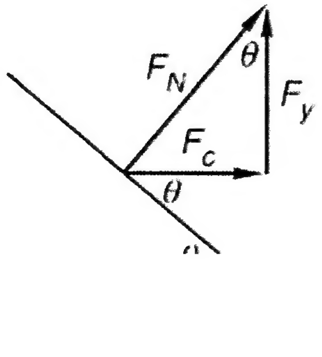
$=$ **12,600 newtons** solved

(b) $F_y = w = mg$ relationship

$= (1400\text{ kg})\left(9.81\ \dfrac{\text{N}}{\text{kg}}\right)$ substituted

$= 13.734$ newtons $\xrightarrow{\text{SD}}$ **13,700 newtons** solved

(c) Now we use the values of 12,600 N for F_c and 13,734 N for F_y and find the required angle of bank.

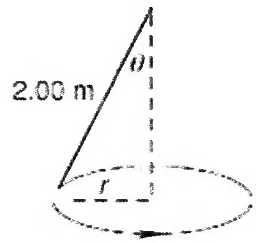


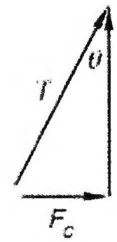
$$\theta = \arctan\frac{F_c}{F_y} = \arctan\frac{12{,}600}{13{,}734} = \mathbf{42.53^\circ}$$

Centripetal Force in Several Settings

7)

A 3.00 kg mass attached to a string 2.00 meters long swings in a circle at 1.20 revolutions per second. Find the tension in the string. Find the angle θ.





The radius of the circle is

$$r = 2.00 \sin \theta$$

and the force necessary to keep the mass moving in a circle is the centripetal force, which is equal to the horizontal component of the tension.

$$F_c = m\frac{v^2}{r} = T \sin \theta$$

Substituting $r\omega$ for v, where $\omega = 1.20\frac{\text{rev}}{\text{s}} \times 2\pi\frac{\text{rad}}{\text{rev}} = 2.40\pi\frac{\text{rad}}{\text{s}}$, we get

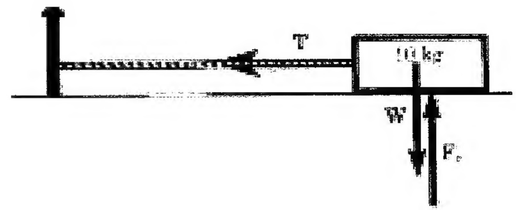
$T \sin \theta = m\dfrac{(r\omega)^2}{r}$	equal forces
$T \sin \theta = mr\omega^2$	simplified
$T \sin \theta = 3.00(2.00 \sin \theta)(2.40\pi)^2$	substituted $r = 2.00 \sin \theta$
$T \cancel{\sin \theta} = 341.1 \cancel{\sin \theta}$	canceled $\sin \theta$
$T = 341.1\text{ N} \xrightarrow{\text{SD}} \mathbf{341\text{ N}}$	solved

To find the angle θ, we set the vertical component of the tension equal to the weight and solve for θ.

$T \cos \theta = mg$	equal forces
$341.1 \cos \theta = 3.00(9.81)$	substituted
$\cos \theta = \dfrac{29.43}{341.1}$	rearranged
$\theta = \mathbf{85.05}$	solved

Centripetal Force in Several Settings

1) A 10-kg block rests on a smooth surface and is attached to a vertical peg by rope. What is the tension T in the rope if the block moves in horizontal circle of radius 2 m at an angular speed of 100 rev/min? First, we find the magnitude of the centripetal acceleration:



$$a_c = r * \omega^2$$

The block is acted on by three forces: its weight, the upward force of the surface, and the tension in the rope (an elastic force). The vertical

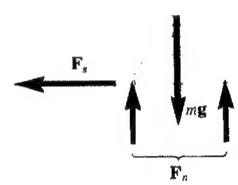
$$\text{net } F = ma$$

$$T = (10 \text{ kg})(219 \text{ m/s}^2) = \boxed{2190 \text{ N}}$$

The direction of T is the same as that of a_c, namely, toward the center of the circular path. We have found the force on the block; by Newton's third law the block exerts an equal and opposite force on the pivot.

2) What is the maximum speed at which a car of mass m can go around an unbanked curve of radius 40 m, if the coefficient of static friction between tires and road is 0.7?

As in the preceding example, the weight of the car mg is balanced by the upward push F_N of the road. The net force is that due to friction, and Fs = 0.7F_N = 0.7 m g, since the car is presumably about to skid and the maximum force of friction is being obtained. To solve we will use the centripetal force equation.



$$F_c = m\frac{v^2}{r} = \mu\ F_N$$

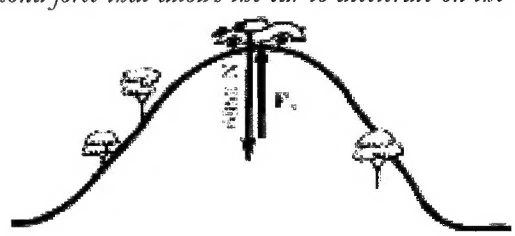
$$\frac{v^2}{40m} = .7 * 9.8\frac{m}{s^2}$$

Notice that the masses cancel which means that the size of the object making a turn is not important, only its speed.

$$v = 16.6\frac{m}{s}$$

3) What is the net force of a 700-kg sports car on the car, as it goes at 10 m/s over the crest of a hill having a radius of curvature of 35 m measured in a vertical plane. Notice that the car is going over a curved surface and that it is moving over the curved surface. In order to travel over the curved surface the car exerts a force equal to its weight and a second force that allows the car to accelerate on the curve. The surface (F_N) likewise exerts an equal and opposite force upon the car.

The car exerts a force of 6800 N down while the surface exerts a force of 8800 N up which accounts for the surface to push up on the car and allows it to be pushed off the surface.



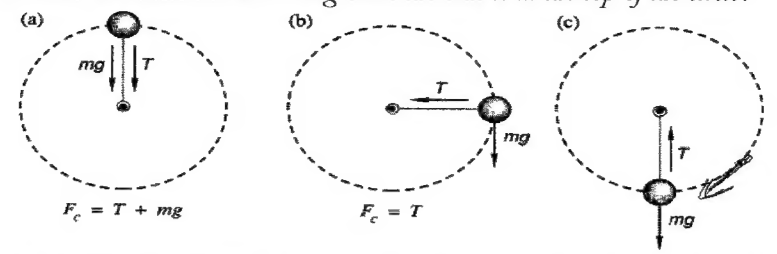
$$F_N + F_c = F_{up}$$

$$m * g + m\frac{v^2}{r} = 700 * 9.8 + 700 * \frac{10^2}{35} = 6800\,N + 2000\,N = 8800N$$

$$F_{down} + F_{Up} = F_{net} \qquad -6800\,N + 8800\,N = 2000N$$

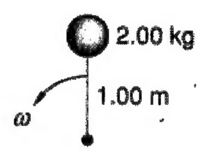
Centripetal Force in Several Settings

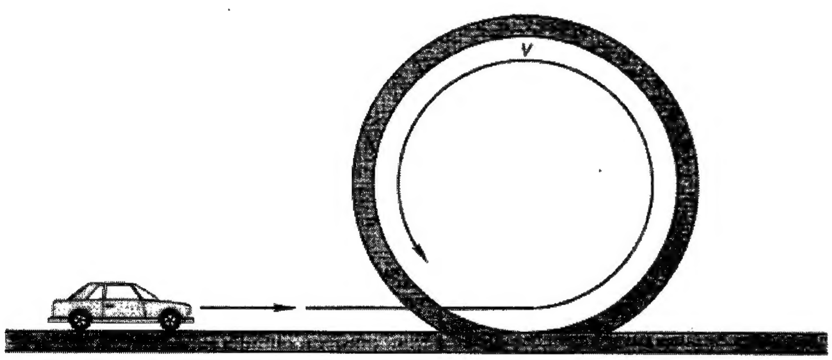
4) John attaches a string 1-meter long to a 2.00-kg ball and swings it in a vertical circle so that its angular speed at the top of the circle is 3.00 revolutions per second. What is the tension in the string when the ball is al the top of the circle?



When a weight of a twirled object is considered, the effect of the weight and centripetal force must be evaluated in order to find the net tension upon the rope. The speed of the weight decreases on the way up and increases on the way down. Since the centripetal force required keeping the eight in a circle depends on the speed, the required force changes as the weight goes around the circle. When the weight is at the tip, as in (a), the weight mg and the tension T act downward as they combine to provide the centripetal force required. When the weight is halfway between the top and the bottom, s in (b) the weight mg acts downward and the tension T acts horizontally. At this point the tension T must provide all of the necessary centripetal force. In (c) when the weight is at the bottom, the tension T must be great enough to overcome the weight mg and still provide the necessary centripetal force to keep the weight moving in a circle.

5) *A toy car has a mass of 3 kg. What minimum speed must it have at the top of the circular track so that it will not fall? The radius of the circular track is .4 meter.*





The speed of the toy tack will change as it moves around the track, but this change in speed does not concern us because we are interested in what happens at the top. The two forces acting on the car are the downward force of the track, F_T, and the weight of the car, mg. The centripetal force pushing the car forward must be great enough to offset the force of gravity pulling the car down. Thus, the minimum speed of the car is found by setting the centripetal force equal to the weight of the car.

weight = centripetal force

$$mg = \frac{mv^2}{r} \rightarrow g = \frac{v^2}{r} \rightarrow v = \sqrt{r * g}$$

$$v = \sqrt{.4 * 9.8} = 1.98\, m/s^2$$